

## Linear and nonlinear behavior in the high-field insulating state of degenerately doped $n$ -type germanium

M. J. Burns, M. W. Meisel, and Huali Li

*Department of Physics and Center for Ultra-Low Temperature Research, University of Florida, Gainesville, Florida 32611*

(Received 18 March 1991)

We report measurements of the electric-field dependence of the conductivity of degenerately doped  $n$ -type Ge in high magnetic fields. At  $T = 100$  mK, the resistivity increases by more than a factor of 1000 in  $H = 8$  T; yet the resistivity remains linear to  $E > 30$  mV/cm. Precision thermometry indicates that the decrease in resistivity with increasing  $E$  is an intrinsic property of the high-magnetic-field state. The data are consistent with hopping with some influence from Coulomb interactions, but are not consistent with Wigner crystallization or charged-density waves.

In recent years the study of magnetic-field-induced metal-insulator transitions has intensified as the possibility of observing collective behavior has been raised.<sup>1,2</sup> The majority of studies in this area have focused on single-valley, narrow-band direct-gap semiconductors such as  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  (Refs. 1 and 2) and  $\text{InSb}$  (Ref. 3) due to the ease with which one can achieve the extreme quantum limit at moderate magnetic fields, i.e.,  $\xi = \hbar\omega_c/E_F > 1$ , where  $\omega_c$  and  $E_F$  are the cyclotron frequency and Fermi energy. If these materials are degenerately doped and placed in a magnetic field at low temperatures, as the magnetic length  $l = \sqrt{\hbar c/eH}$  becomes comparable to the donor impurity spacing, a transition from metallic to insulating behavior takes place. Considerable controversy exists on the exact nature of this high-field insulating state. One view is that electron-electron interactions dominate the high-field state, causing the electrons to minimize their electrostatic repulsive energy by forming an ordered lattice, a so-called Wigner crystal. The other view is that under these high-field conditions, the donor binding energies possess a strong magnetic-field dependence as the electronic wave functions are compressed around the impurity. In the strong-field limit, ( $\gamma = \hbar\omega_c/2E_D \gg 1$ ), the magnetic field dominates the binding energy ( $E_D$ ) and causes the electrons to strongly localize onto the impurities. Traditionally, this idea is referred to as "magnetic freeze-out" and is strictly limited to single-particle binding onto isolated donors.

In addition to the aforementioned  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  and  $\text{InSb}$ , other semiconducting systems also can show a magnetic-field-induced metal-insulator transition, namely Ge. A multivalley, relatively-large-indirect-band-gap semiconductor, Ge shows such a transition at modest magnetic fields ( $\xi \sim 1$ ,  $\gamma < 1$ ). Uncompensated, degenerately doped  $n$ -type Ge undergoes a transition from metallic to insulating behavior in its resistivity but shows very little change in the Hall effect.<sup>4,5</sup>

Studies of resistivity,<sup>1-3</sup> the Hall effect,<sup>1-3</sup> and optical properties<sup>6,7</sup> in  $\text{InSb}$  and  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  have usually been interpreted as providing evidence for either magnetic freeze-out on donors,<sup>3,6</sup> freeze-out of a metallic impurity band,<sup>7</sup> or collective phenomena such as charge-density

waves (CDW's) or Wigner crystals,<sup>1,2</sup> although Mani<sup>8</sup> has recently questioned whether there is a distinct transition at all. The random impurity potential should tend to suppress the formation of any ordered charged structure such as a Wigner crystal. If a Wigner crystal phase were to form, it would have to adjust itself to fit about the random impurity potential, building up considerable strain over large distances. Over large enough distances, such strains would exceed the yield stress of the Wigner crystal causing it to break into domains. As a result, any Wigner crystal would be a collection of crystallites (polycrystalline) rather than a single crystal over the sample volume. This view would only be valid if the "size" of a Wigner crystallite,  $L$ , is greater than the mean electron spacing, otherwise the system would lose its long-range order and would best be viewed as a rigid electron "gas" or "electron glass." The electron-impurity and electron-electron interactions are of equal strength in zero magnetic field, but one would expect the compression of the impurity wave functions with increasing magnetic field to break this symmetry and enhance the electron-impurity interaction. So even if the zero-field random impurity potential is not strong enough to prevent the formation of a Wigner crystal, the magnetic field will accentuate the impurity potential and, hence, may tend to destroy it at high enough fields.

One of the proposed means of detecting the presence of a Wigner crystal or charge-density wave is based on the idea that the disorder potential impedes the ability of the crystal to slide collectively in response to an externally applied electric field. Such a "pinned" Wigner crystal would only display collective motion when the external field could supply enough energy to "slide" it over the random impurity potential, giving rise to non-Ohmic resistivity and the generation of narrow band noise.<sup>9</sup> In traditional charge-density-wave systems, such depinning manifests itself as Ohmic behavior of the conductivity when the collective state is pinned and a nonlinear increase in conductivity as the collective state slides. Field and co-workers,<sup>10,11</sup> based on low-frequency  $I$ - $V$  curves, have reported the onset of nonlinear conduction suggestive of a sliding collective state at  $\sim 0.6$  mV/cm in  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  at 10 mK and in 6.4 T. This paper reports

our investigations of whether similar behavior would occur in the high-field insulating state of a different metal-insulator system, namely degenerately doped  $n$ -type germanium.

Samples of Czochralski-grown nominally uncompensated, degenerately doped (with Sb)  $n$ -type Ge with carrier densities above the critical density ( $n_c = 1.5 \times 10^{17} \text{ cm}^{-3}$ ) were cut from the same boule as the samples in Ref. 12, which was tested at low temperatures for impurity striations as described in Ref. 5. Using the method of Thomas *et al.*,<sup>13</sup> we estimate the compensation of our samples to be below 5%. The samples were cut with all faces along the  $\langle 100 \rangle$  axes and etched in 3:1 (by volume) concentrated  $\text{HNO}_3$  to HF solution. Electrical contacts were made by using Sn saturated with Sb, and typical contact resistances were 2–4  $\Omega$  at low temperatures. The samples were mounted on one end of an oxygen-free high-conductivity copper rod that was bolted to the mixing chamber of a home-made  $^3\text{He}$ - $^4\text{He}$  dilution refrigerator and were cooled to below 100 mK. For increased thermal contact between the sample and the copper tail, one current lead to the sample was grounded to the rod to allow a low-resistance electrical-thermal path between sample and mixing chamber. In addition to the mixing chamber thermometry, a low-mass (2.6 mg)  $\text{RuO}_2$  chip resistor thermometer that had previously been calibrated to 30 mK and in magnetic fields up to 8 T was mounted on the sample holder next to the sample.<sup>14</sup> Changes as small as 0.05 mK could be detected in the  $\text{RuO}_2$  chip resistor thermometer and the absolute temperature determined to better than 0.1 mK.

Standard  $I$ - $V$  curve tracing methods involve sweeping a dc field or placing a small ac modulation on a swept dc electric field. These methods require a continuous electric field (duty cycle=100%). In traditional charge-density-wave systems, the threshold electric fields needed to depin the charge-density wave can be larger than the fields where heating becomes important using standard  $I$ - $V$  curve tracing methods. In order to minimize this problem, we used a four-probe pulsed technique that allowed us to reduce the power dissipation in the sample by over a factor of 100. Voltage pulses of 4 ms duration were passed at 1.1 s intervals through the sample, thus probing the sample with a duty cycle of less than 0.4%. The voltage across a resistor in series with the current leads of the sample, and the voltage response of the sample were measured by a digital oscilloscope that averaged over several pulses (up to 256). Checks were made of the sample behavior as a function of pulse width to verify that the widths were long enough to prevent spurious results due to any possible frequency dependence of the conductivity. The  $\text{RuO}_2$  thermometer was continuously monitored on a sensitive bridge for indications of lattice heating of the sample. The duty cycle was also varied as an additional check for sample heating.

In Fig. 1 we show the resistivity of a sample with  $n \sim 2.6 \times 10^{17} \text{ cm}^{-3}$  as a function of magnetic field to 8 T for temperatures down to 100 mK. The mobility at room temperature for this sample was  $610 \text{ cm}^2/\text{V s}$ . The resistivity data of Fig. 1 was measured using conventional low-noise ac ( $\sim 25 \text{ Hz}$ ) lock-in amplifier techniques. The

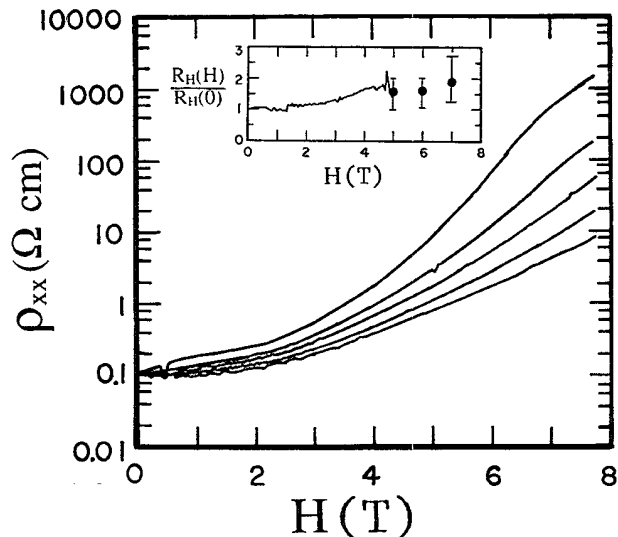


FIG. 1. The resistivity of the sample as a function of magnetic field at (top to bottom curves) 100, 200, 300, 400, and 500 mK. The inset shows the normalized Hall coefficient as a function of field at 100 mK. The resistivity increases by over three orders of magnitude at 100 mK while the Hall coefficient exhibits little variation.

inset shows the Hall effect at 100 mK, measured using the five-contact Hall bar geometry (virtual Hall contact) combined with magnetic-field reversals as described in Ref. 5. The resistivity shows a rapid rise when the resistivity crosses the Mott minimum metallic conductivity<sup>15</sup> for Ge:Sb at  $(0.143 \text{ } \Omega \text{ cm})^{-1}$ . Extrapolation<sup>4,16,17</sup> of the temperature dependence of the conductivity at various magnetic fields to zero temperature, indicates the  $T=0$  conductivity would be zero at  $H_c \cong 4 \text{ T}$ , consistent with Refs. 4, 5, and 12. The Hall effect shows little change throughout the transition also consistent with Refs. 4, 5, and 12, and similar to that reported in Si metal-oxide-semiconductor field-effect transistor structures.<sup>18–20</sup> It should be noted that at 4 T for this system  $\gamma \sim 0.2$  and  $\xi \sim 2$ , in contrast to  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  (Refs. 1 and 2) and InSb (Ref. 3) where the transitions occur in a “high-field” limit<sup>4</sup> of  $\gamma \gg 1$  and  $\xi \sim 5–10$ .

In Fig. 2 we show the sample resistivity as a function of applied electric field, normalized to the zero-electric-field limit at 100 mK for two magnetic-field values, 6 and 8 T. The onset of nonlinear conduction is approximately 60 mV/cm at both fields. In Fig. 2, it is important to note that, within the noise, the resistivity remains Ohmic for almost two orders of magnitude in electric field. At 6 and 8 T, the zero-electric-field resistivity is 3 and 4.5 orders of magnitude above the 0 T values, thus placing the sample well into the insulating state. At the highest electric fields probed, the  $\text{RuO}_2$  thermometer mounted next to the sample indicated a temperature rise in the sample of less than 8 mK. Since the samples were probed using voltage pulses, the power dissipation ( $V^2/R$ ) in the samples increases as the resistivity drops, so it is not unreasonable to expect a sample to thermally deviate slightly from the mixing chamber temperature when entering the nonlinear regime. However, at the largest electric-

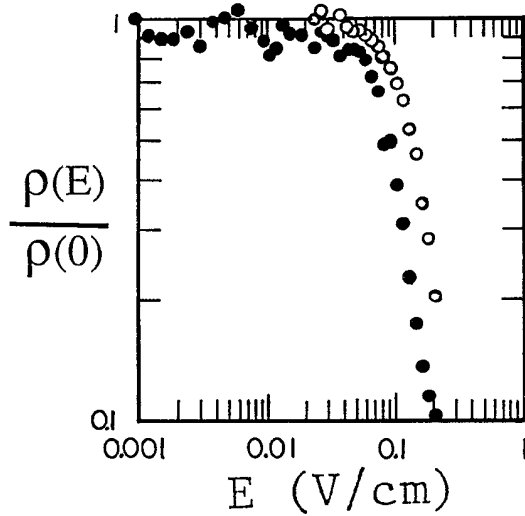


FIG. 2. Electric-field dependence of the sample resistance at 8 T (solid circles) and 6 T (open circles) at 100 mK. Decreasing the magnetic field only very slightly increases the electric field at which non-Ohmic behavior is observed.

field values probed, there is a decrease in sample resistance of about a factor of 10. Comparison with Fig. 1 indicates that if the high-electric-field resistance decrease of Fig. 2 were due merely to sample heating, temperature rises of  $\sim 100$  mK at 8 T and  $\sim 200$  mK at 6 T are required. Therefore the major non-Ohmic effect displayed in Fig. 2 is due to the nonlinear response of the high-field insulating state.

In Fig. 3 we show the sample resistivity as a function of applied electric field, normalized to the zero-electric-field limit for two temperatures, 100 and 300 mK. As the sample temperature is increased, the electric field required for non-Ohmic conduction increases. If we assign the onset of non-Ohmic conduction to be when the resistivity has unambiguously departed from the zero-field value, we would assign a threshold field  $E_0$  of  $\sim 30$  mV/cm at 100 mK and  $\sim 70$  mV/cm at 300 mK.

One would expect a Wigner crystal or CDW state to have differing threshold electric fields at differing values of the CDW order parameter with increasing magnetic field, similar to that seen with an increasing CDW order parameter in conventional CDW systems,<sup>21</sup> and unlike the behavior seen in Fig. 2. However Figs. 2 and 3 do appear consistent with transport arising from single-particle hopping. We can make a crude estimate of the electric field required for significant deviations from Ohmic behavior for both noninteracting<sup>22</sup> (Mott) and interacting<sup>23</sup> (Efros) variable range hopping. The electric field inserts a directional dependence into the hopping probability through the hopping energy,  $E_H$ , in that

$$E_H \rightarrow E_H - \frac{e\mathbf{E} \cdot \mathbf{r}}{kT}$$

In other words, the probability of hopping with the field is enhanced over that of hopping against the field. For moderate fields,<sup>24</sup> when  $E \leq kT/(eR\xi)$ , the conductivity is multiplied by

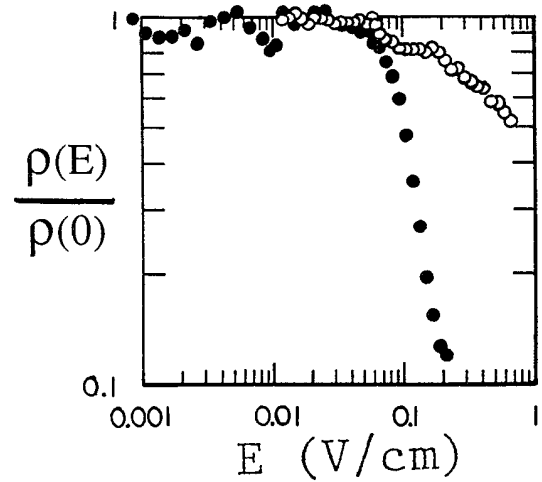


FIG. 3. Electric-field dependence of the sample resistance at 100 mK (solid circles) and 300 mK (open circles) at 8 T. Increasing the temperature increases the field at which non-Ohmic behavior is observed.

$$\exp \left[ \frac{eRE\xi}{kT} \right],$$

where  $\xi \approx 0.17$  in 3D,  $R$  is the low field hopping distance given by

$$\left[ \frac{3a^*}{4\pi D(E_F)2kT} \right]^{1/4}$$

and

$$\left[ \frac{e^2 a^*}{2\kappa kT} \right]^{1/2}$$

in the Mott and Efros models,  $D(E_F)$  is the density of states at the Fermi level, and  $\kappa=16$  is the dielectric constant of Ge. Non-Ohmic conduction should take place when  $E \approx E_0 = kT/(eR\xi)$ . In very high fields, where  $E \gg E_0$ , the conductivity becomes temperature independent and follows an electric-field dependence<sup>24-26</sup>

$$\sigma \sim \exp \left[ \frac{E}{E_0} \right]^{1/S},$$

( $S \approx 2$  or  $4$ ), which has recently been observed in GaAs.<sup>27</sup>

We can make a crude estimate of  $E_0$  with a few simple assumptions. For a system with a donor density of  $N_D$ , the density of states can be assumed to be of the order of  $N_D/E_F$  for  $\gamma < 1$ . We may assume the spatial extent of the localized wave functions that form in the high-field state to be slightly less than, but of the order of, the average impurity spacing, i.e.,  $a^* \sim n^{1/3}$ , a reasonable assumption since the sample maintains charge neutrality on a macroscopic scale and yet the impurities are no longer forming extended states. For the data shown in Figs. 1 and 2, this estimate gives values for the onset of non-Ohmic behavior in the high-magnetic-field state of  $E_0$  on the order of 1 and 5 mV/cm for the noninteracting (Mott) model at 100 and 300 mK, respectively. A similar

estimate indicates  $E_0$  is on the order of 40 and 207 mV/cm for the interacting (Efros) model at 100 and 300 mK. Although these estimates are very crude, they do illustrate that, as interactions become important, the external electric fields are required to move the electrons in a *correlated* fashion. The stronger the interactions, the greater the amount of correlated motion. If the correlations were periodic, the system would be a CDW or Wigner crystal. At higher temperatures, both the Mott and Efros hopping models indicate that the onset of non-Ohmic behavior is *increased* to higher-electric-field values since larger thermal energies enhance the ability of an electron to hop against the field. A larger electric field is then needed to force all the hops to be down field. For a CDW or Wigner crystal, raising the temperature weakens the crystal, lowering its ability to support a shear force, thereby *decreasing* the field for depinning.<sup>21</sup> The behavior displayed in Figs. 2 and 3 is clearly more consistent with a high-magnetic-field hopping state in which interactions are important rather than with a sliding

Wigner crystal or CDW state.

In conclusion, we have driven degenerately doped  $n$ -type Ge through the metal-insulator transition, which shows a large resistivity increase but no substantial change in the Hall coefficient. We have observed non-Ohmic resistivity which is consistent with variable range hopping in the presence of Coulomb interactions, which might eventually produce some correlations in the hopping, but whose temperature dependence would rule out strongly collective ground states such as a Wigner crystal or CDW.

We thank S. E. Brown, D. Romero, and C. J. Stanton for helpful discussions. This work was supported in part by the University of Florida Division of Sponsored research (M.J.B. and M.W.M.), a grant by the U.S. Defense Advanced Research Projects Agency—Office of Naval Research (M.J.B., Grant No. MDA-972-88-J-1006), and the Florida High Technology and Industry Council (M.W.M.).

- 
- <sup>1</sup>B. Schlicht and G. Nimtz, in *Physics of Narrow Gap Semiconductors*, edited by E. Gornik, H. Heinrich, and L. Palmetshofer, Springer Lecture Notes in Physics Vol. 152 (Springer-Verlag, Berlin, 1982), p. 383; J. P. Stadler and G. Nimtz, *Phys. Rev. Lett.* **56**, 382 (1986).
- <sup>2</sup>T. F. Rosenbaum, S. B. Field, D. A. Nelson, and P. B. Littlewood, *Phys. Rev. Lett.* **54**, 241 (1985).
- <sup>3</sup>M. Shayegan, V. J. Goldman, H. D. Drew, D. A. Nelson, and P. M. Tedrow, *Phys. Rev. B* **32**, 6952 (1986).
- <sup>4</sup>R. M. Westervelt, M. J. Burns, P. F. Hopkins, and A. J. Rimberg, in *Anderson Localization*, edited by T. Ando and H. Fukuyama (Springer-Verlag, Berlin, 1988), p. 33.
- <sup>5</sup>P. F. Hopkins, M. J. Burns, A. J. Rimberg, and R. M. Westervelt, *Phys. Rev. B* **39**, 12 708 (1989).
- <sup>6</sup>V. J. Goldman, H. D. Drew, M. Shayegan, and D. A. Nelson, *Phys. Rev. Lett.* **56**, 968 (1986).
- <sup>7</sup>J. B. Choi and H. D. Drew, *Phys. Rev. B* **41**, 8229 (1990).
- <sup>8</sup>R. G. Mani, *Phys. Rev. B* **40**, 8091 (1989).
- <sup>9</sup>G. Grüner, A. Zawadowski, and P. M. Chaikin, *Phys. Rev. Lett.* **46**, 511 (1981).
- <sup>10</sup>S. B. Field, D. H. Reich, B. S. Shivaram, T. F. Rosenbaum, D. A. Nelson, and P. B. Littlewood, *Phys. Rev. B* **33**, 5082 (1986).
- <sup>11</sup>S. B. Field, D. H. Reich, T. F. Rosenbaum, P. B. Littlewood, and D. A. Nelson, *Phys. Rev. B* **38**, 1856 (1988).
- <sup>12</sup>M. J. Burns, *Phys. Rev. B* **40**, 5473 (1989).
- <sup>13</sup>G. A. Thomas, Y. Ootuka, S. Katsumoto, S. Kobayashi, and W. Sasaki, *Phys. Rev. B* **25**, 4288 (1982).
- <sup>14</sup>M. W. Meisel, G. R. Stewart, and E. D. Adams, *Cryogenics* **29**, 1168 (1989).
- <sup>15</sup>N. F. Mott, *Philos. Mag.* **26**, 1015 (1972); **29**, 613 (1974); N. F. Mott and E. A. Davis, *Electronic Properties in Non-Crystalline Materials*, 2nd ed. (Oxford University Press, Oxford, 1979).
- <sup>16</sup>M. C. Maliepaard, M. Pepper, R. Newbury, and G. Hill, *Phys. Rev. Lett.* **61**, 369 (1988).
- <sup>17</sup>M. C. Maliepaard, M. Pepper, R. Newbury, J. E. F. Frost, D. C. Peacock, D. A. Ritchie, G. A. C. Jones, and G. Hill, *Phys. Rev. B* **39**, 1430 (1988).
- <sup>18</sup>E. Arnold, *Appl. Phys. Lett.* **25**, 705 (1974).
- <sup>19</sup>M. Pepper, *Philos. Mag. B* **38**, 515 (1978).
- <sup>20</sup>J. P. Thompson, *Philos. Mag. B* **38**, 527 (1978).
- <sup>21</sup>For a review, see G. Grüner, *Rev. Mod. Phys.* **60**, 1129 (1988).
- <sup>22</sup>N. F. Mott, *J. Non-Cryst. Solids* **1**, 1 (1968).
- <sup>23</sup>A. L. Efros and B. I. Shklovskii, *J. Phys. C* **8**, L49 (1975).
- <sup>24</sup>M. Pollak and I. Reiss, *J. Phys. C* **9**, 2339 (1976).
- <sup>25</sup>B. I. Shklovskii, *Fiz. Tekh. Poluprovodn.* **6**, 2335 (1972) [*Sov. Phys. Semicond.* **6**, 1964 (1973)].
- <sup>26</sup>R. Rentzsch, H. Berger, and I. S. Shlimak, *Phys. Status Solidi A* **54**, 487 (1979).
- <sup>27</sup>F. Tremblay, M. Pepper, D. Ritchie, D. C. Peacock, J. E. F. Frost, and G. A. C. Jones, *Phys. Rev. B* **39**, 8059 (1989); F. Tremblay, M. Pepper, R. Newbury, D. Ritchie, D. C. Peacock, J. E. F. Frost, G. A. C. Jones, and G. Hill, *Phys. Rev. B* **40**, 3387 (1990).