Quick & Dirty Review of Resistivity Measurement Techniques

by Dr. Michael J. Burns*

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Resistivity Measurement

Kelvin Probe Resistance Measurements/4-Point Probe Resistance Measurement Techniques

The term "Kelvin Probe Resistance Measurement" is simply synonymous with the term 4-Point Probe Resistance Measurement. The term "4-Point Probe Resistance Measurement" is generally preferred by Physicists and Material Scientists and is the term we will use below. Both terms are by themselves so imprecise as to be meaningless on their own. To complicate matters, Kelvin Probe Resistance Measurement is also often confused with Kelvin Bridge measurement[1], which itself like saying "Wheatstone Bridge Measurement". In other words, a Kelvin Bridge measurement only describes a specific bridge circuit[2] without regard to sample measurement issues.

Within the category of 4-Point Probe Resistance measurements, there are 4 major types of sample resistance measurement techniques with very specific assumptions. The only common assumption between the variations is the assumption that the sample be homogeneous to prevent voltage nesting. All of these variations of resistance measurements can be used for both chordal resistance ($R_{ch} = V/I$) and dynamic resistance

* Present Address: Boston College Physics, 140 Commonwealth Ave., Chestnut Hill MA 02467

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($R_{dy} = \partial V/\partial I$) measurements. For materials research, intrinsic resistivity is always assumed to be measured at low enough excitations for the chordal and dynamic resistances to be the same, even in highly non-linear materials. In other words, in the limit of infinitesimally small sample excitation.

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In-Line Spreading Resistance Measurement[3]

In-Line Spreading Resistance measurements are the most common resistance technique used in the semiconductor industry. This is the technique used in pogo-contact commercial sheet resistance measurement units commonly found in semiconductor fabrication facilities.

Assumptions

1. *Sample Homogenious*
2. *Sample Isotropic*
3. *Sample two-dimensional (thickness small compared to breadth & width)*
   And as usually practiced in the semiconductor industry:
4. *Probe spacing small compared to sample breadth & width*

In the In-Line Spreading Resistance method, four electrical contacts are made in a line on the sample to be measured. As generally practiced, the spacing of the 4 contacts (s) relative to each other is very small compared to length (a) and width (d) of the two-dimensional sample (see the inset of the correction factor figure below). This $s<<a,d$ aspect is not actually a general requirement for the technique to be valid.

The excitation current is applied between the outer two contacts, and the voltage drop measured between the inner two. From their ratio, a resistance is computed. The sheet resistively ($\rho_\square$) in ohms per square of the sample can then be computed using this resistance along with a correction factor CF based on s, a and d.

$$\rho_\square = CF\ R_{sp}$$

As generally practiced with the spacing of the 4 contacts (s) relative to each other very small compared to length (a) and width (d) of the two-dimensional sample, this correction factor simply becomes $CF=\pi/ln2$, where $ln2$ is the natural logarithm of 2 (~0.693147…) and π is 3.141593…. The ratio $\pi/ln2$ is ~4.5.

$$\rho_\square = \frac{\pi}{ln2}\ R_{sp} \approx 4.5\ R_{sp}\ (\text{If } s<<a,d)$$

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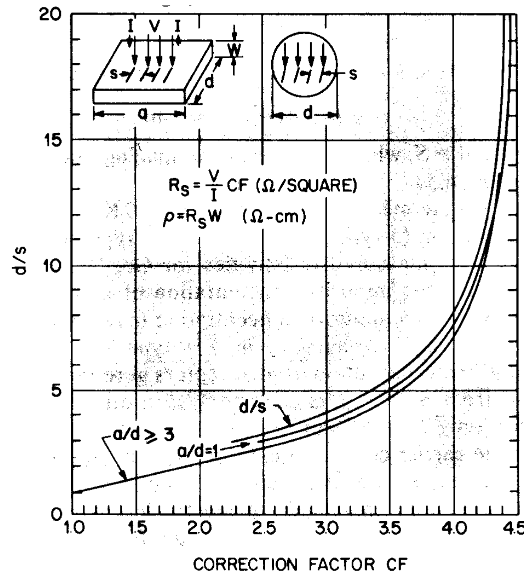

In the figure above[4] we show how in the general case *CF* depends on *a*, *s*, and *d*. It is intuitively obvious why the correction factor asymptotically approaches a constant as $s<a,d$ without much sensitivity to either the ratio of a/d or to the individual values of *a* and *d*, once one recognizes that the current distribution between the current contacts follows the same pattern as the electric field lines of a dipole in a two dimensional space with a moment length of 3*s*. So once one is a distance a few times 3*s* away from the contacts, the current distribution has dropped to essentially zero. So portions of the sample further away than roughly this distance simple do not contribute to the measurement.[5]

The sheet resistivity can be converted to a bulk resistivity by multiplying by the thickness of the sample *W*, resulting in the bulk resistivity in terms of the In-Line Spreading Resistance R_{sp}, and sample thickness *W* of

$$\rho = CF\ W\ R_{sp}$$

or in the case as usually practiced where the contact spacing is small compared to the sample size ($s<<d,a$):

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$$\rho = \frac{\pi}{ln2} W R_{sp} \approx 4.5\ W R_{sp}\ (\text{If } s<<a,d)$$

The reason the In-Line Spreading Resistance technique is so popular is precisely because it can be exploited as a *local* probe of the *local* resistivity in a larger sample simply by making s small. Another advantage is that knowledge of the actual values of s, a and d never enter into the computation of the resistivity, and so never need to be known in any way *if* one is operating in the limit of $s<<d,a$. In this limit, the only geometrical parameter needed to be known with any precision to compute the bulk resistivity is the thickness W. By probing systematically with very small contact spacing s over a sample, one can create a map of the sample resistivity. Exploiting this allows one, for example, to probe the uniformity of the sheet resistivity of a film deposited across a large wafer, thus giving one information allowing prediction of ones tolerance at making resistors using that material on an identical wafer. That is precisely why the In-Line Spreading Resistance technique, in the limit of $s<<d,a$, is the most common resistance measurement technique used in the semiconductor industry.

Use of $\rho = 4.5\ W R_{sp}$ instead of the more general $\rho = CF\ W R_{sp}$ with $CF < 4.5$ in a geometry when $s \approx a,d$ is the most common measurement mistake encountered with the In-Line Spreading Resistance method.

Van der Pauw Resistance Measurement[6]

Van der Pauw resistance measurements is the second most common resistance technique used in the semiconductor industry after spreading resistance measurements.

Assumptions

1. *Sample Homogenious*
2. *Sample Isotropic*
3. *Sample two-dimensional (thickness small compared to breadth & width)*
4. *Sample boundary sharply defined*

The Van der Pauw method exploits the reciprocity theorem of electromagnetism, which states that the current in a detector divided by the voltage at the source remains constant when source and detector are interchanged. In the Van der Pauw method, four

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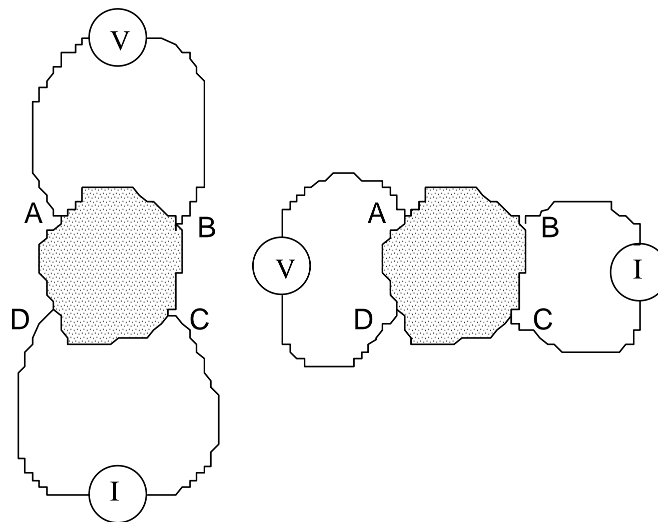
electrical contacts are made on the perimeter of the sample to be measured. This means that the perimeter has to be sharply defined. The contacts cannot be on the edge of a region where the conductivity is merely dropping below a high value to a lower value, nor can they be placed substantially inside the edge of the sample. If the sample can be patterned to restrict a central commonly connected region, such as in a 4-leaf clover pattern, then contacts can be placed anywhere on the peninsulas and the "sample" is really just the region connecting the peninsulas.[7]



The contact arrangements for the two resistance measurements, $R_{ab\text{-}cd}$ and $R_{bc\text{-}da}$ used in the Van der Pauw resitivity measurement.

Once the four contacts are established, two resistance measurements ($R_{ab\text{-}cd}$ and $R_{bc\text{-}da}$) are taken with the difference being that one each of adjacent current and voltage contacts are switched, as illustrated in the figure above. The sheet resistively ($\rho_\square$) in ohms per square of the sample can then be computed using these two resistances along with a correction factor based on their ratio.

$$\rho_\square = \frac{\pi}{ln2} \frac{R_{ab\text{-}cd} + R_{bc\text{-}da}}{2} f$$

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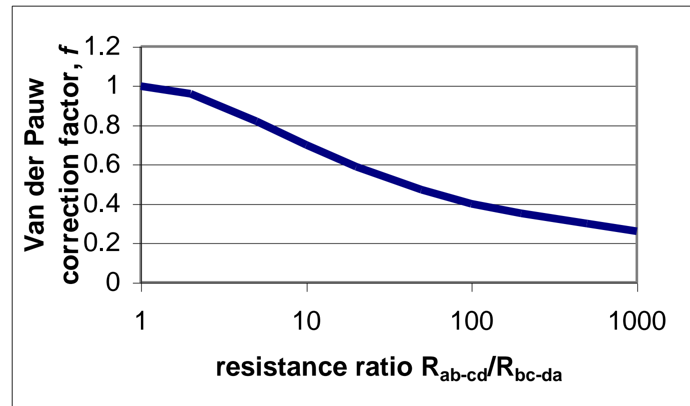
where *ln*2 is the natural logarithm of 2 (~0.693147…) and π is 3.141593…. The ratio π/*ln*2 is ~4.5. The correction factor *f* is shown below as a function of the ratio of $R_{ab\text{-}cd}$ and $R_{bc\text{-}da}$.



Correction factor based on the ratio of the two resistance measurements, $R_{ab\text{-}cd}$ and $R_{bc\text{-}da}$ used in the Van der Pauw resitivity measurement.

The sheet resistivity can be converted to a bulk resistivity by multiplying by the thickness of the sample, resulting in the bulk resistivity in terms of the Van der Pauw measures resistances $R_{ab\text{-}cd}$ and $R_{bc\text{-}da}$, and sample thickness *W* of

$$\rho = \frac{\pi}{ln2} W \frac{R_{ab\text{-}cd} + R_{bc\text{-}da}}{2} f$$

It should be noted that the ratio of the two measurements depends purely upon geometry not upon any inhomogeneity or isotropy in the material being measured. If the material or film is not homogeneous and isotropic, or the boundary of the sample is not sharp as may be encountered when measuring thin films whose characteristics vary with position, the Van der Pauw method is not valid and will as a general rule err by giving a resistivity that is too large (too resistive). Violation of the isotropic and homogeneity requirements is the most commonly found misapplication of the Van der Pauw method. For homogeneous isotropic samples, placing of contacts in the middle of a larger film is the most common measurement mistake encountered with the Van der Pauw method.

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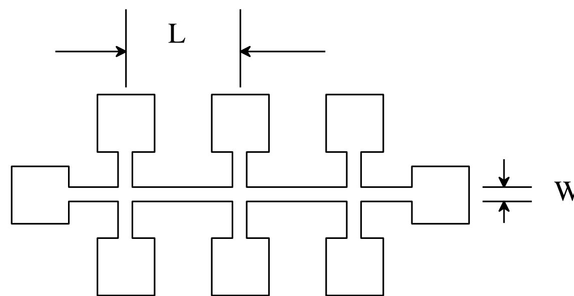
Hall Bar Resistance Measurement[8]

Hall Bar Resistance measurements are commonly used when both resistance and carrier concentration are required. [9]

Assumptions

1. *Sample Homogenious*
2. *Sample Isotropic*
3. *Sample length be long compared to thickness & width*
4. *Sample boundary sharply defined*



The contact arrangements for the Hall Bar measurements. Current is injected through the left and right most contacts, and voltages are measured on the side contacts.

The Hall Bar arrangement is commonly used in situations where films are patterned before measurements. With a Hall bar arrangement, both resistivity and, if a magnetic field is available, Hall Effect can be measured. From these two measurements, carrier concentration and carrier mobility can be computed.

In the Hall Bar arrangement, the resistance is computed as the voltage between contacts a length L apart down the strip of width W, divided by the current through the two end contacts I, $R=V/I$. The sheet resistivity is then

$$\rho_{\square} = \frac{W\,R}{L}$$

if the film thickness is t, then the bulk resistivity is

$$\rho = \frac{Wt\,R}{L}$$

One aspect of the Hall Bar measurement is that it assumes the current density down the central strip is uniform. Particularly in magnetic fields, this assumption can be

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violated in samples that have only moderate length to width and thickness ratios. How long is long enough depends on the magnetoresistance of the sample.[10] As a general rule, one wants $L/W >\sim 7$, and $W >> t$. This is the most common measurement mistake encountered with the Hall Bar method.

Montgomery Resistance Measurement[11]

Assumptions

1. *Sample Homogenious*
2. *Sample boundary sharply defined*

This method is a generalization of the Van der Pauw method to release it from the assumptions of two dimensionality and the assumption of isotropy. Again, it exploits the reciprocity theorem of electromagnetism but in this case for an anisotropic medium which can be treated using simple scaling relations between the principle axii. Because the sample can be anisotropic and does not have to be 2 dimensional, often this technique is exploited in studies of anisotropic organic conductors such as charge-density-wave and spin-density-wave charge transfer salts[12]. Since we did not need to utilize this technique in the MICE program, it is mentioned and the cardinal literature on this technique sited for completeness, but the Montgomery Method will not be described here.

Voltage Nesting

Voltage nesting is the major issue those inexperienced or unwary in resistance measurements face. Voltage nesting arises when the assumption of sample homogeneity breaks down. In the crudest examples, this occurs if the sample cracks but it can also arise if contacts freeze out, as can happen at cryogenic temperatures, can happen if inhomogeneities cause local changes in resistivity that vary by orders of magnitude, or as can happen in semiconductors if the sample has doping striations of high and low conductivity. In the latter, such striations are known to form in Czochralski grown semiconductors at high doping concentrations.[13]

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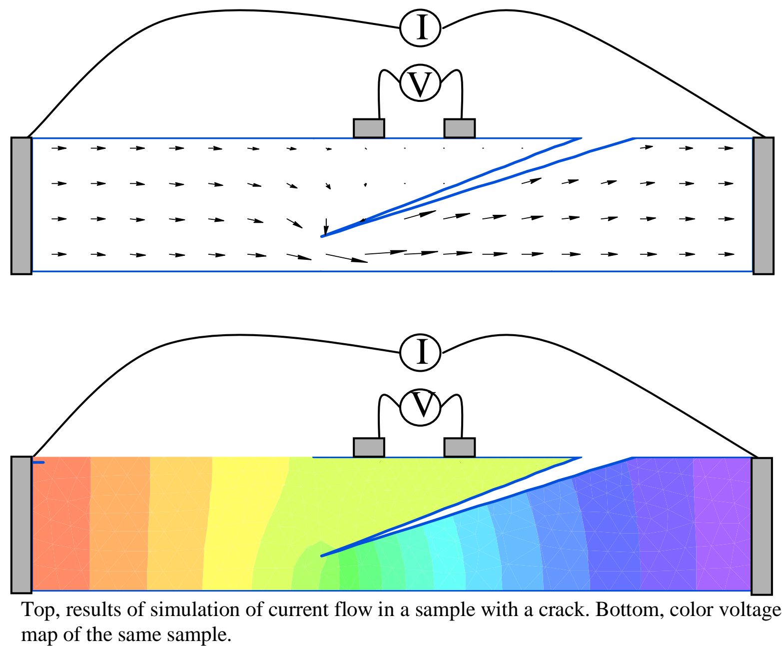

Top, results of simulation of current flow in a sample with a crack. Bottom, color voltage map of the same sample.

The figure above illustrates a course grid finite-element simulation of an extreme situation of a crack in an otherwise homogeneous sample. The top figure shows the current density in the sample as a function of position. The size of the arrows is proportional to the local current density. The bottom figure shows the results of the same simulation but the results presented as the local electrical potential (voltage) relative to the current injection electrode. The peninsula with the two voltage contacts has no current flow in it resulting in it becoming an equipotential surface. An uncritical evaluation of the data would indicate zero voltage response for the sample despite current flow, or zero resistance. This specific voltage nesting example would produce erroneous data falsely indicating a superconducting sample even when the resistivity is far above zero. More complicated inhomogeneity patterns can result in arbitrary voltage responses, or even compete reversals of polarity falsely indicating negative chordal resistance (V/I)[xiv], not to be confused with the real physical possibility of a truly negative dynamic resistance ($\partial V/\partial I$), as can be seen in tunnel diodes.[xv] Voltage nesting from inhomogeneous samples is the single most common error for producing incorrect resistivity measurements in electronic materials development programs after misapplication of the various Kelvin probe methods.

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[9] K. Seeger "Semiconductor Physics, 3rd edition" (Springer-Verlag, Berlin, 1985) page 67
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[12] F. Wudl, E. Aharon-Shalom, D. Nalewajek, J. V. Waszczak, W. M. Walsh, Jr., L. W. Rupp, Jr., P. M. Chaikin, R. Lacoe, M. J. Burns, T. O. Poehler, M. A. Beno, and J. M. Williams, "Ditetramethyltetraselenafulvalenium Florsulfate: The Effect of a Dipolar Anion on the Solid State Physical Properties of the $(TMTSF)_2X$ Phase," *J. Chem. Phys.* ***76***, 5497 (1982)
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[xiv] Shoukai Wang and D.D.L. Chung, "Apparent Negative Electrical Resistance in Carbon Fiber Composites", *Composites: Part B **30**,* 579–590 (1999).
[xv] J.M. Ziman, "Principles of the Theory of Solids, 2nd edition", (Cambridge University Press, Cambridge, 1972) p 195

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